Student-generated mathematical modeling lessons as a final project

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Abstract

Mathematical modeling is a process for describing the world with mathematics, as well as a practice for learning mathematics in the K-12 classroom. In this study, I implemented modeling curricula in a capstone mathematics course for secondary teacher preparation. Students drew on the diversity of contexts and math concepts in the modeling tasks to create their own mathematical modeling lessons as a final project assessment. I found that students relied on the components of rich mathematics tasks to select the topic, math content, and structure of their tasks. In particular, they drew on the structure of The Sprinkler Task to generate authentic situations and connect to their funds of knowledge. In this paper, I utilize a project-based learning goals framework to illustrate the benefits of incorporating modeling tasks in undergraduate curricula for mathematics teacher preparation.

Keywords: Mathematical Modeling; Initial teacher preparation; Project-based learning; Higher education

Background on Mathematical Modeling

Mathematical modeling is "a process in which students consider and make sense of an everyday situation that will be analyzed using mathematics for the purpose of understanding, explaining, or predicting something" (Anhalt et al., 2018, p. 202). In the Common Core State Standards in the U.S., the standard for mathematical practice *Model with Mathematics* states that "mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (CCSSI, 2010, p.7). Due to the cyclic structure of the modeling process, the revision of mathematical work is emphasized as a justification tool for formulating a realistic solution to an authentic problem (Anhalt et al., 2018). Mathematical modeling tasks benefit students in many ways, such as connecting multiple content

areas and developing problem-solving skills and creativity (e.g., Tidwell et al., 2021; Mousoulides et al., 2008).

Modeling tasks are productive activities for future teachers, as modeling requires knowledge of relevant mathematical content and demonstration of mathematical processes and practices (AMTE, 2017). Secondary teachers are expected to facilitate modeling tasks, and their students are expected to "learn how to model the world with math", so there needs to be a better understanding of how prospective secondary teachers engage in modeling tasks as learners and future teachers. Modeling tasks require a different way of teaching and facilitating (Doerr, 2007) and can be challenging for teachers who are new to modeling; thus, it is crucial to include experiences with modeling in teacher preparation courses (Anhalt & Cortez, 2016; Anhalt et al., 2018). However, mathematical modeling is less commonly included in teacher preparation courses (Tidwell et al., 2021).

Research on mathematics teacher preparation has shown that creating rich mathematics lessons that are rooted in authentic situations is challenging (e.g., Aguirre et al., 2013; Turner et al., 2022), especially when the lesson contexts are meant to align with students' lived experiences and knowledge (Civil, 2002). Civil explored how children's mathematics learning and engagement can be better und3erstood through the lens of *funds of knowledge*, as well as how mathematics activities can draw on students' cultural knowledge and experiences to help them connect school mathematics to authentic situations. Similarly, mathematical modeling tasks can be designed to draw on students' funds of knowledge to help them connect classroom mathematics to authentic situations in their school, family, home, and community (Bennett & Neihaus, 2022).

While there are few examples of secondary preservice teachers generating rich mathematics lessons that connect to their lived experiences, there are several studies conducted with elementary preservice teachers, such as elementary preservice teachers creating problem-solving lessons that connect to students' funds of knowledge (Aguirre et al., 2013) or elementary preservice teachers creating modeling problems for young children (Paolucci & Wessels, 2017). In particular, Aguirre and colleagues (2013) found that preservice teachers felt a tension between addressing the mathematics content and the authentic situation in a meaningful way and were inclined to address one or the other. In sum, there are documented challenges for preservice teachers when generating mathematical modeling tasks, especially when they are expected to attend to multiple components within a created task.

Methods

The research questions I address in this paper are: 1) How can modeling tasks be used as "rich tasks" in a project-based mathematics course for secondary pre-service teachers? and 2) What modeling tasks and structures did PSTs draw on when designing their own tasks?

The data presented here are part of a larger qualitative study. This study took place at a large, public, research-oriented university in the U.S. Participants included 11 secondary pre-service teachers in one section of a capstone mathematics course focused on mathematical modeling for secondary teacher preparation. During this semester, I piloted modeling curricula

from the MODULE(S^2) project and selected several tasks that drew on geometry concepts. These students had no introduction to mathematical modeling prior to this course.

Students were introduced to the modeling process and solved modeling tasks in small groups throughout the semester. Weekly tasks focused on specific math content areas and realistic topics. Additionally, they varied in terms of their *math concepts* (e.g., measuring distance and other geometric properties, graphing data and finding trends, and generalizing patterns and justifying equations), *task topics* (e.g., lack of resources, improving on or resolving a social injustice, and predicting future data), and *task structures* (e.g., map-based, choosing the best option from a given list, and using a news or research article with authentic data event).

For their final project in the course, students were prompted to create their own modeling tasks based on the weekly tasks that we had experienced together. To support students in this process, I created scaffolds and guidelines to help students focus on important aspects of modeling and problem posing. I directed students to analyze three components of the tasks they solved (i.e., the math concepts, task topics, and tas3k structures) and make connections to their generated modeling tasks. In this way, I assessed students' understanding of the modeling process and specific mathematics concepts through the modeling tasks that they created.

Data Collection and Analysis

Data collection consisted of students' final project submission, which took the form of a "scaffolding assignment", poster, oral presentation, and written reflection. In the scaffolding assignment, students explicitly stated which modeling they drew on when creating their own task. There were 10 projects since two students worked as a pair. I also collected students' solutions and modeling reports for the 13 tasks assigned throughout the semester. Most of the 13 tasks came from the MODULE(S^2) project curricula. Four of these lessons over eight class sessions were video recorded. However, in this paper I focus on the analysis of the final projects. Data analysis consisted of artifact analysis (Hatch, 2002) on the final modeling projects.

For an analytic framework, I adapted Stolk & Martello's Project Goals Framework (2018) to explore how students drew on the topics, task structures, and math concepts of modeling tasks from class. In particular, I focused on their responses in the scaffolding assignment, and explored how students incorporated elements of "rich tasks" and the modeling process into their tasks. Project-based learning increases student learning and cognition of course content and addresses skills such as creativity, critical thinking, disciplinary integration, communication, and collaboration (Stolk & Martello, 2018). Indeed, these transferable skills are measurable via Stolk and Martello's Project Goals Framework. By using this framework, the effectiveness of project-based assessments can be measured in teacher preparation courses.

Findings

Six of the 10 final projects drew on a modeling task called the Sprinkler task. This was the most frequently cited task by students, but they drew on this task for different reasons.

The Sprinkler task prompt states that a neighbor has asked for help to purchase and set up a low-maintenance watering system for her backyard and garden. Students were provided with a diagram of the backyard on grid paper and a list of available watering equipment (with prices) from a local gardening store. Students had to create a watering plan, and in doing so, decide what features were most important, such as covering the entire yard, conserving water, or saving money. The task connected to math concepts such as the Pythagorean Theorem, equations of circles and other geometric shapes, rigid transformations, areas of irregular and overlapping shapes, and a coordinate system suggested by the scale grid on the backyard diagram. The model was the plan and procedure for determining which sprinklers to use and how, where, and in what order to place them. The model determined the mathematics concepts that were used, based on the priority for watering. For a more detailed description of the task and supplemental materials, see Bennett and Neihaus (2022) and Neihaus and Bennett (2022).

Recall that students completed a total of 13 modeling tasks throughout the semester, yet six of 10 projects were inspired by the Sprinkler task. Why did a majority of students draw on the Sprinkler task? Students shared that they appreciated the math concepts and task structure from the Sprinkler task, yet they did not mention the topic of the task. (See Table 1 for their quotes.)

Table 1

Students shared how they used elements of the Sprinkler Task in their created tasks.

	Quotes from students about how the Sprinkler Task was useful
1	"I found inspiration in the Sprinkler Task regarding their providing a list of different materials that could be used to water the garden and giving more choice to those who are completing the task to decide what would be best to purchase/ what is the most cost-effective option. This relates to the mission of [my task] because the goal is to also find the most cost-effective method to complete a task, only instead of fully watering a garden, it is for traveling from point A to point B."
2	"One thing that I really liked about this task was it started with a problem […] The fact that this was a real-life problem that needed to be solved really intrigued me. Another thing that I liked about this task was we were given a handout. Being able to work hands-on with a paper and write on it and measure things is something I want to do with my final project. I also want to be able to put my final project on a grid like we did here in the Sprinkler Task. This allows you to measure distance or area so much easier."
3	"I would say [my] model is most similar to the Sprinkler Task. This task is essentially asking for the most optimal set up and specific list of option with reasoning why. This model will be me picking through a list of options[…] and deciding which ones will do the job the best."
4	"Another reason why I am using the Sprinkler Task is how it used geometry. The Sprinkler Task made students utilize their geometry skills to be efficient with water coverage. Students will be required to also use geometry skills to know the most effective shapes at holding water."
5	"In the Sprinkler Task, students often had to create circles to model the area that each sprinkler would be able to water. I would like [students] to use similar concepts of circles to estimate the amount of land area and people that would be benefitted from an NFL [American football franchise] team placed in various locations."
6	"The other task I foresee referencing is going to be the Sprinkler Task. This task is similar to my own in the sense that both are very wide open in how to approach them."

As students mentioned, they found the Sprinkler task to be a helpful example due to its wide range of mathematics concepts; multiple approaches, strategies, and solutions; and, multiple components, such as using a diagram and graphing it in Desmos, using a list of options, making calculations, researching information about watering, and connecting to their personal experiences and funds of knowledge. Essentially, this task attended to many of the features of a rich mathematics task, so there were many opportunities for students to draw on it. An additional reason students may have focused on this task is that, as a class, we engaged in transparent conversations about how the task was created from a real-life experience and how we anticipated using it for teaching secondary students. The future teachers were asked to reflect on the Sprinkler task afterward, so they may have been more aware of the elements that went into designing this task.

Figure 1 presents an area proportional Venn diagram (with the exception of the Topic circle, which should have area zero) to illustrate how the student quotes from Table 1 were mapped onto the math concepts or task structure categories. For the six students who drew on the Sprinkler task, the structure of the task was helpful to four students and the mathematics concepts were relevant for three students. None of these six students connected the topic of the Sprinkler task to their own, even though they chose this task out of 13 possible modeling tasks to inspire and guide their own task creation.

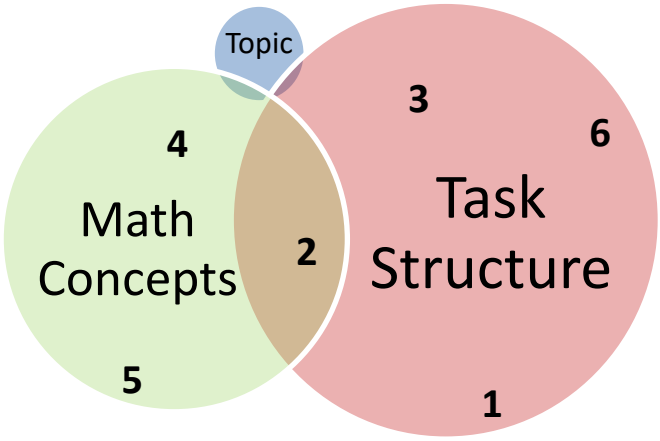

Figure 1. An area proportional Venn diagram of the Sprinkler task elements that students drew on.

Further analysis of the math concepts, topics, and task structures revealed other themes across all 10 final projects. The two most frequent task structure elements used were a map-based structure (6 projects) and choosing the best option from a given list (5 projects). The most common topic was a situation with a lack of resources in a given area, such as a food desert with insufficient grocery stores or a city that needs a new school (3 projects). Both of the task structures mentioned above were components of the Sprinkler task.

Discussion and Conclusion

Overall, students connected to the modeling tasks in the course that addressed real problems and social injustices, such as a lack of resources in a community. However, they did not necessarily use those topics in their own generated tasks, but rather, drew on the task

structures and math concepts utilized in those tasks. As preservice teachers create their own tasks that align with the modeling process, they make problem-posing decisions related to context, math concepts, structure, and attention to issues of social justice (see also, Jung & Magiera, 2021). The findings of this study demonstrate that students pay at least as much attention to the structures of the task as they do the context, which is noteworthy when exploring modeling with future teachers (Anhalt & Cortez, 2016). The topics of mathematics tasks should be interesting and relevant, but there should also be a variety of task structures, representations, and mathematics concepts. This holds implications for how educators design and use contextually realistic tasks in mathematics courses for future teachers, which is crucial for developing teachers as reflective practitioners (Sevinc & Lesh, 2021).

It is important for pre-service teachers to experience alternate forms of assessment, especially in summative situations (Prasad, 2020). The Project-based Learning Goals framework (Stolk & Martello, 2018) highlighted the specific components of the final project that were most supportive for students. Projects in mathematics courses need specific goals, and these goals should be transparent and measurable. As students analyze the tasks they create, they can also use the project-based framework as a resource to see which goals they attended to.

Educators have long written about the importance of problem-posing, both for math learners and math teachers (e.g., Cai & Hwang, 2021). The beginning phases of the math modeling process encourage problem posing and creating a simplified version of a complex issue. Demonstrating to students how to examine a task's topic, structure, and math concepts allowed them to pose mathematical questions in the context of a rich task. Scaffolding the assignments up to the final project helped students with problem posing and creating a task that they can implement with their future students.

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