
# Discrete Mathematics in the School Curriculum: A Curricular Definition

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**Abstract**

Discrete mathematics has been recommended for inclusion in the school curriculum for more than 30 years. A persistent problem has been to establish a universally agreed-upon definition of discrete mathematics. No attempt is made here to present such a mathematical definition, but instead a *curricular definition* is given. This definition is adapted from the recent rigorous survey paper by Sandefur, et al. (2022). In this brief paper here, I will present and elaborate this definition, provide rationale for it, and give some brief examples.

*Keywords*: Discrete Mathematics; Mathematics education; Pre-university education; Curriculum; Modeling.

## Introduction

Discrete mathematics has been recommended for inclusion in the school curriculum for more than 30 years (e.g., Hart 1985, NCTM 1989, Hart and Martin 2018, Sandefur, et al. 2022). One barrier to implementing this recommendation has been the difficulty of crafting a universally agreed-upon mathematical definition of discrete mathematics. I propose here, instead, a *curricular definition*. This definition is based on the "curricular characterization" given in the recent thorough survey paper by Sandefur, et al. (2022). As such, it is a definition that is consistent with professional experience and literature. Most importantly, such a curricular definition is needed to focus the work of teachers, curriculum developers, and researchers, as they work to reinvigorate the implementation of discrete mathematics into the school curriculum.

## Curricular Definition of Discrete Mathematics

In Sandefur, et al. (2022), we give the following curricular characterization of discrete mathematics.

> Out of the diverse range of discrete mathematics topics and related mathematical competencies, we propose a focused, practical characterization of discrete mathematics, aimed at clarifying discrete mathematics for the school curricula. .... This leads to the following *curricular characterization of discrete mathematics in the schools*, in terms of key themes, topics, and distinctive mathematical competencies:
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> - *Key themes* that can be addressed through discrete mathematics: networks, enumeration, sequential change, strategic decision making, fairness, and the Internet.
> - Specific discrete *topics* that address these themes: graph theory, combinatorics, iteration and recursion, game theory, the mathematics of voting and fair division, discrete games and puzzles, and number theory including some mathematics of information processing such as coding and cryptography.
> - *Distinctive mathematical practices* emphasized in the teaching and learning of discrete mathematics: recursive thinking, combinatorial reasoning, algorithmic problem solving, discrete optimization, and, above all, discrete mathematical modeling.
>
> (Sandefur, et al. 2022)

These themes, topics, and practices can be refocussed as a curricular *definition* of discrete mathematics, which can then provide a much-needed explicit "target" for teachers, curriculum developers, and researchers. Each of these themes, along with the associated topics and practices, will now be discussed.

## Networks: Graph Theory–Modeling Relationships and Connections–Optimization–Algorithmic Problem Solving

There are many relevant problems that involve relationships and connections, such as prerequisite or conflict relationships, or connections in transportation, communication, or social networks. Diagrams of vertices and edges can be used to model such problems. These diagrams are sometimes called *graphs*, *networks*, or *vertex-edge graphs*. When the edges have an indicated direction, then the diagram is called a *directed graph* or *digraph*.

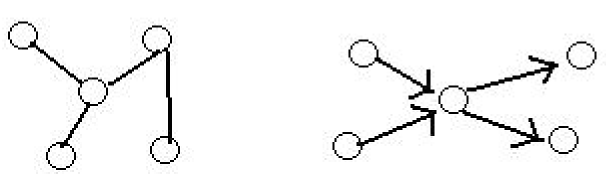

Figure 1: vertex-edge graph on the left, digraph on the right

For example, vertices might represent towns in the rainforest, edges represent roads between some of the towns, and "weights" on the edges could be the cost to lay fiber optic Internet cable along a given road. Then a typical problem is to find a network that provides a fiber optic Internet connection for all towns but with the lowest total cost. A solution to this problem is called a *minimal spanning tree*. There are intuitive algorithms (systematic step-by-step procedures) for finding a minimal spanning tree. For example, sequentially choose the shortest edge available without creating any circuits. A key part of discrete mathematics is solving problems by thinking about algorithms, then analyzing those algorithms in terms of whether they always work and if they are efficient. The version of a "shortest-edge" algorithm stated just above will in fact always work and is efficient. However, if you change the algorithm just slightly, so that you sequentially choose the shortest edge *from where you're at* (called the nearest-neighbor algorithm), instead of choosing the shortest edge available *anywhere,* without creating a circuit (Kruskal's algorithm), then the algorithm will not always yield a solution.

## Enumeration: Combinatorics–Combinatorial Reasoning

Many problems involve answering the question, "How many?", such as how many lottery numbers are possible, how many ways a voter can rank candidates in a ranked-choice voting election, or how many routes are possible for a salesperson to visit all their clients. Such problems are solved by a particular type of reasoning we can call *combinatorial reasoning*. Specific mathematical tools in this domain include combinations, permutations, the fundamental principle of counting, and the pigeon-hole principle. Overall, this area of mathematics is called combinatorics. This is an area in discrete mathematics that is already often found in school curricula, although it is often focused more on formulas and less on the more useful aspects of combinatorial reasoning and modeling.

## Sequential Change: Iteration and Recursion–Recursive Thinking

As the saying goes, change is the only constant in life! A particular common kind of change is sequential step-by-step change, such as how the amount you owe on a loan changes month by month as you make payments, or how the amount of medicine in your system changes as you take your regular doses. Iteration and recursion can be used to model such change. For example, if you take a 20mg dose of an antibiotic every 6 hours and your body eliminates 20% every 6 hours, then the amount in your system over 6-hour intervals can be modeled by an equation like NEXT = 0.8 * NOW + 20. This is called a recursive equation (or recurrence relation or difference equation). Such *recursive thinking* provides a very accessible way to model problems that have more complicated closed-form equations, and allows natural use of technology like spreadsheets to analyze the problems.

## Strategic Decision Making: Game Theory

Strategic decision making in competitive situations often involves the mathematics of games, called game theory. This area of mathematics was primarily developed by von Neumann and Morgenstern in the 1940s, and its first major applications were in helping to make strategic decisions in World War II. Other applications include analyzing the famous "prisoner's dilemma," or game shows on TV, or the game between a goalkeeper and a penalty-taker in football where one player must decide where to kick and the other must decide where to guard, or corporate "games" where business decisions must be made strategically with competitors in mind. The analysis of such games begins by just applying logical thinking, then proceeds to use elementary algebra and probability, and thus is accessible and relevant for school students.

## Fairness: The Mathematics of Voting and Fair Division

The idea of fairness is everywhere in life, from getting your fair share of birthday cake to fairly apportioning seats in parliament and ensuring fair elections. Analyzing such problems involves the mathematics of voting and fair division. For fair voting, since the goal is to reflect the will of the people, it makes sense to get more information about voters' preferences through voting by ranking the candidates, not just by choosing your favorite. This is called ranked-choice voting. There are many ways to analyze the results of ranked-choice voting, requiring only arithmetic and logical thinking, such as assigning points for preferences (common in sports and entertainment voting), or doing an "instant" runoff (whereby the runoff happens instantly based on the ranked choices voters made when they voted). Regarding fair division, again just using arithmetic and logical thinking, there are intuitive algorithmic methods to fairly divide cake or land or an inherited estate, or fairly apportion seats in congress or parliament.

## The Internet: Mathematics of Information Processing–Cryptography

Of course, the Internet is an ubiquitous part of modern life. How much mathematical understanding of the Internet would be useful for most school students? That's a hard question and one we'll figure out with time, but an initial answer would be that at least some understanding would be useful, perhaps some understanding of privacy and security on the Internet. This involves cryptography. In particular, a brief unit on public-key cryptography, used for example in e-commerce and "https" Internet URLs, is interesting, accessible, involves reasoning about modular arithmetic, and has potential to improve students' attitudes about the usefulness of mathematics and improve their skill in mathematical modeling and reasoning about number systems.

## Conclusion

There have been recommendations for many years to include discrete mathematics in the school curriculum. But progress has been slow. One reason for this is the lack of an explicit "target." We need to clarify the curricular goal when trying to implement discrete mathematics into the curriculum. We propose a curricular definition, or characterization, of discrete mathematics that provides such a goal. In terms of the school mathematics curriculum, we can define discrete mathematics in terms of key themes, topics, and mathematical competencies:

- Key themes of discrete mathematics – networks, enumeration, sequential change, strategic decision making, fairness, and the Internet
- Discrete mathematics topics that address these themes – graph theory, combinatorics, iteration and recursion, game theory, the mathematics of voting and fair division, discrete games and puzzles, and number theory including some mathematics of information processing such as coding and cryptography
- Distinctive mathematical competencies emphasized in the teaching and learning of discrete mathematics – recursive thinking, combinatorial reasoning, algorithmic problem solving, discrete optimization, and, above all, discrete mathematical modeling

It is evident from the key themes that discrete mathematics is relevant to students' lives in contemporary society. The topics that address those themes are important mathematical topics and, crucially, very accessible to school students. The mathematical competencies are powerful processes and habits of mind that will serve students well, beyond just in mathematics classes. This explicit characterization of discrete mathematics for the school curriculum, a "curricular definition" if you will, is intended to provide a focus that will help teachers, curriculum developers, and researchers as they work to strengthen and modernize the school curriculum by incorporating discrete mathematics.

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