



Teaching for Robust Understanding: Powerful instruction for all Students

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Abstract

We want students to emerge from our classrooms being mathematically knowledgeable and resourceful thinkers and problem solvers, with a sense of personal agency and positive dispositions about themselves as mathematics learners. The question is, how do we get there? That is, what kinds of learning environments help all students become powerful mathematical thinkers? The Teaching for Robust Understanding (TRU) framework says what counts. If the content is rich; if students are engaged in sensemaking and productive struggle; if there are ways to engage every student with core mathematical content and practices; if norms support engagement that supports a sense of agency, ownership over the content, and a sense of mathematical self; and if there are robust feedback mechanisms (formative assessment), then good stuff happens.

Key words: Leaning; Powerful instruction; Rich mathematics; Cognitive Demand; Equitable access; Mathematical identity; Formative assessment

Introduction

We all want to create classrooms from which students emerge as powerful and agentic mathematical thinkers. The students should know a lot of mathematics – not just content, but also practices such as reasoning, problem solving, and mathematizing. They should have positive mathematical dispositions and identities, seeing themselves as capable and actively using their mathematical understandings, both inside the classroom and in the “real world.” This is a two-fold challenge. The first challenge is the complexity of teaching itself. There are so many things to attend to when we teach. Which ones make a difference, which ones should we attend to? The second is, how can we get better at those things?

The Teaching for Robust Understanding (TRU) Framework addresses these challenges. The TRU framework identifies five key dimensions of mathematics classrooms. Research indicates that the degree to which students emerge from instruction being powerful thinkers is directly related to the degree that instruction does well along the five dimensions of TRU. The challenge, then, is to improve instruction along the five TRU dimensions.

The TRU research group (see <https://truframework.org/>) has developed numerous tools to help professional learning communities work together, in sustained ways, to reflect on and improve their teaching practices. We have just published two books aimed at helping teachers understand and implement the ideas in TRU. *Helping Students Become Powerful Mathematics Thinkers: Case Studies and Methods on Teaching for Robust Understanding* (Schoenfeld, Fink, Zuñiga-Ruiz, Huang, Wei, & Chirinda, 2023) presents a series of case studies of classroom teaching. It engages readers in conversations about what teachers might do, and how different choices might affect what students learn. *Mathematics Teaching on Target: A guide to Teaching for Robust Understanding at all Grade Levels* (Schoenfeld, Fink, Sayavedra, Weltman, & Zuñiga-Ruiz, 2023) shows how to enrich tasks and activities so that students can interact more deeply with rich mathematics.

This paper introduces the TRU Framework and then illustrates our tools for professional development. It works quickly through one case study from the first book and showing how the ideas in the second book can be used to enrich classroom activities.

The Teaching for Robust Understanding (TRU) Framework

There is a great deal of research on things that matter in instruction. The problem is, there is too much. (If you do a google search for “improving teaching,” google will report “about 2,830,000,000 results!”) Long lists of suggestions are of little value: Teachers don’t have the time to try dozens of suggestions. Given that there are so many, how would they know which to try, and what to focus on?

Our intention was to distill the literature – which literally had hundreds and hundreds of ideas for improving teaching – into a collection of categories with the following properties:

- Every category represents an important dimension of instruction
- Each dimension can be the meaningful focus of professional development
- Together, the dimensions focus on everything that is important
- There is a small number of dimensions, so it is possible to keep them all in mind during planning, teaching, and reflecting on lessons.

After numerous attempts, we found a clustering that was *coherent* – but, did the categories in it actually represent aspects of teaching we could see in practice; were they meaningful; but, would it be *useful*? This was an empirical question. Could we identify each dimension in practice? Could we say how effective the teaching was in that dimension? And, how could we test the hypothesis that doing well in various dimensions would result in students learning more?

The way to test these ideas is to develop a scoring rubric. Videotapes of teachers who were known to be excellent provided examples of the kinds of teaching we wanted to see. After a number of attempts we were able to develop a scoring rubric that responded to variation – lessons that felt richer scored higher – and in which higher scores corresponded to classroom activities valued in the literature on teaching. Later, using a database that included videos of classrooms and records of student scores on tests of thinking and problem solving, we were able to show that, the better a classroom did on the five dimensions of TRU, the more students learned. (Schoenfeld, Floden, and the algebra teaching study and mathematics assessment projects, 2018) Other studies, e.g., Prediger & Neugebauer 2021, replicated and extended these findings. At a theoretical level, we can be confident in saying that the framework reliably identifies the important dimensions of classroom activity.

Figure 1 presents a schematic description of the TRU Framework. The better a classroom does in every dimension, the more students will learn.

The Five Dimensions of Powerful Mathematics Classrooms				
The Mathematics	Cognitive Demand	Equitable Access to Mathematics	Agency, Ownership, and Identity	Formative Assessment
<i>The extent to which classroom activity structures provide opportunities for students to become knowledgeable, flexible, and resourceful mathematical thinkers. Discussions are focused and coherent, providing opportunities to learn mathematical ideas, techniques, and perspectives, make connections, and develop productive mathematical habits of mind.</i>	<i>The extent to which students have opportunities to grapple with and make sense of important mathematical ideas and their use. Students learn best when they are challenged in ways that provide room and support for growth, with task difficulty ranging from moderate to demanding. The level of challenge should be conducive to what has been called “productive struggle.”</i>	<i>The extent to which classroom activity structures invite and support the active engagement of <u>all</u> of the students in the classroom with the core mathematical content being addressed by the class. Classrooms in which a small number of students get most of the “air time” are not equitable, no matter how rich the content: all students need to be involved in meaningful ways.</i>	<i>The extent to which students are provided opportunities to “walk the walk and talk the talk” – to contribute to conversations about mathematical ideas, to build on others’ ideas and have others build on theirs – in ways that contribute to their development of agency (the willingness to engage), their ownership over the content, and the development of positive identities as thinkers and learners.</i>	<i>The extent to which classroom activities elicit student thinking and subsequent interactions respond to those ideas, building on productive beginnings and addressing emerging misunderstandings. Powerful instruction “meets students where they are” and gives them opportunities to deepen their understandings.</i>

Figure 1. The five dimensions of powerful mathematics classrooms

The first dimension concerns the quality of the mathematical content and practices that the students encounter. Mathematics should be coherent and connected. Students should see connections between underlying ideas and procedures they use. They should have opportunities for mathematical sense-making, reasoning, and problem solving. If these opportunities are present in the classroom, then there is the potential for significant learning. If they are not, then it is unlikely that students will learn them. The quality of the mathematics students experience establishes an upper bound on what they will learn.

But, the fact that high-quality mathematics is being discussed is no guarantee that students will learn it! I have attended many lectures where the mathematics appeared to be beautiful, but I didn't understand it at all. Mathematics learning is a form of sense-making. To learn, you need to be able to connect what you know to the things you are studying. This is dimension 2 of TRU, Cognitive Demand. If what you are working on is too easy, there is nothing to learn. But if what you are working on is so far from what you know that you cannot make process, then meaningful learning will not take place. The idea is to engage in “productive struggle.” The central question for Dimension 2 is, how much opportunity is there for sense-making and productive struggle?

Dimension 3 concerns Equitable Access to mathematics. In an equitable classroom, *every* student has the opportunity to engage meaningfully with the important mathematical ideas in the lesson. The question for Dimension 3 is, how many students have such opportunities?

Dimension 4 concerns students' mathematical identities. Some students think they are bad at mathematics. They don't enjoy it, avoiding mathematics when they can. Others enjoy mathematics. They jump into opportunities for problem solving and for thinking things through. They have a sense of initiative, or mathematical agency. When they have made sense of an idea, they have a sense of ownership of that idea. They have positive mathematics identities. The question is, to what degree does the classroom provide every student with opportunities to engage with mathematics so that they can develop positive mathematics identities?

Finally, Dimension 5 concerns formative assessment. When instruction works well, student thinking is made public and instruction responds. If students are not challenged, they are given more challenging tasks. If students are simply using formulas, they are asked why the formulas work. If students are lost, they are given some hints – not so much that challenge is removed, but enough to enable them to engage in productive struggle. The questions for Dimension 5 are, how responsive is instruction to student thinking? Can responsiveness be improved?

Most frameworks for discussing teaching focus primarily on the teacher. TRU is different. Since our interest is in how instruction supports student learning, the fundamental question in TRU is: How does the student experience instruction? That is, TRU involves a fundamental shift in perspective, from teacher-centered to student-centered. The key question is *not*: “Do I like what the teacher is doing?” It is: “What does instruction feel like, from the point of view of the student?” Figure 2 summarizes this emphasis. Our research and development are devoted to helping teachers make students' answers to the questions in Figure 2 as rich as possible.

There is a great deal of evidence for the validity of the TRU Framework, that teachers can learn to teach in ways that are increasingly aligned with Figures 1 and 2, that when instruction aligns with TRU, students learn a great deal. See Prediger & Neugebauer 2021; Schoenfeld 2013, 2014, 2015, 2016, 2018; Schoenfeld et al., 2019a,b).

Observe the Lesson Through a Student's Eyes	
The Content	<ul style="list-style-type: none"> • What's the big idea in this lesson? • How does it connect to what I already know?
Cognitive Demand	<ul style="list-style-type: none"> • How long am I given to think, and to make sense of things? • What happens when I get stuck? • Am I invited to explain things, or just give answers?
Equitable Access to Content	<ul style="list-style-type: none"> • Do I get to participate in meaningful math learning? • Can I hide or be ignored? In what ways am I kept engaged?
Agency, Ownership, and Identity	<ul style="list-style-type: none"> • What opportunities do I have to explain my ideas? In what ways are they built on? • How am I recognized as being capable and able to contribute?
Formative Assessment	<ul style="list-style-type: none"> • How is my thinking included in classroom discussions? • Does instruction respond to my ideas and help me think more deeply?

Figure 2. Observe the lesson through a student's eyes.

Once my research group was confident that TRU could be helpful, we collaborated with school districts and professional developers to create TRU-related tools that teacher learning communities could use. Two new books will be published in June 2023. The rest of this paper describes what the books offer, both for learning about rich mathematics classrooms and for improving classroom materials and activities. There is only space here to outline the most important ideas. In my conference presentation I will give examples.

Helping students become powerful mathematics thinkers: Case Studies and Methods of Teaching for Robust Understanding

We do not underestimate the challenges of what we call “ambitious and equitable” teaching – teaching that provides *all* students opportunities to engage with, and learn, rich mathematics. It takes years to develop the understandings and skills that are needed. And, of course, the process never stops. Our growth as teachers comes from continuous reflection on our teaching. Although this kind of reflection can be done by individuals, results tend to be more powerful when reflection is done collectively in a professional learning community, in which teachers and others work together to plan instruction, teach, reflect on what happened, and revise accordingly.

Our first new book, *Helping students become powerful mathematics thinkers: Case Studies and Methods of Teaching for Robust Understanding* (Schoenfeld, Fink, Zuñiga-Ruiz, Huang, Wei, & Chirinda, 2023), provides a comprehensive overview of the TRU framework and a number of tools to help teachers do this. The book starts with a deep description of the TRU

framework, explaining what the five TRU dimensions mean in practice, and why they are important. The main part of the book consists of three case studies of instruction. Each case looks closely at about 20 minutes of instruction. Our intention is not to critique the instruction, but to “problematize” it – to examine the options the teacher faces, what possibilities there are for teacher and students, and to think about what the outcomes of different pedagogical decisions might be – not just for mathematics learning, but for all five dimensions of TRU. The experience is like saying, “suppose we could analyze the lesson in ‘slow motion.’ Let’s think about (a) the implications of what the teacher and students did do, and (b) what other actions the teacher might have taken, and the implications of those actions.”

Imagine stopping the video of a lesson at a particularly interesting point, or reading a transcript of the lesson up to that point. We see what has taken place; let us explore the implications. We explore all five dimensions.

Consider Dimension 1, The Mathematics: Can we imagine the kinds of mathematics that the students will engage in from this point on? Is it likely to be richer and more connected? Or, did the interactions seem to narrow the space, so that the mathematics is not likely to be as rich as we would hope? Are there other things the teacher or students might have done or said, that would open things up mathematically, or spark richer explanations, or...?

For Dimension 2, Cognitive Demand: Do the students seem to be working in the “zone of proximal development” (Vygotsky 1978)? Are they engaged in productive struggle? What did the teacher do to address this issue, and how do we think the teacher’s choice might play out? If the tasks seem too easy, what options might there be to make the students’ work richer and more challenging? If the students seem “lost,” are there ways to help them grapple with the problem without telling them what to do or giving hints that remove the challenge completely?

Here is a brief example of the kind of dialogue we have with readers. In one of our case studies, a group of students is clearly confused when the teacher comes to their table. After discussing what they have done, the teacher leaves the students to struggle, without giving them any hints about which direction to pursue. This raises many questions to think about. Will they get frustrated? If they don’t make progress, will this be bad for their mathematical identities? Or, if they are successful, will that be good for their identities, since they know they made sense of something challenging? We don’t have definitive answers – our goal is to illustrate the ongoing dilemmas of teaching, and the possible consequences of particular decisions. In the case study, we get to follow the students after the teacher leaves them, to see how the teacher’s decision actually played out.

For Dimension 3, Equitable Access: Do we see all students engaged with the core mathematics in the lesson? If not, how might that be addressed? What did the teacher do; what might we do in a situation like that? Next time, might we re-design the task so that there are more ways to approach it, or more connections to be made? If two students find different solutions, asking the students to compare and contrast their solutions can enrich the mathematics. This is a much more interesting situation than having a problem for which there is only one way to obtain a solution.

For Dimension 4, Agency, Ownership, and Identity: What opportunities are there for students to venture ideas, to discuss them with others? How do students respond to each other? Are the conversations collaborative? Do students question and build on each other's ideas? What do we think the consequences of the interactions might be? Are there alternative ways to frame student interactions, or ways to modify the activities, so more students can engage productively?

For Dimension 5, Formative Assessment: Is student thinking made public? In what ways? Are there questions one can ask, or instruction the teacher can give, that will make more student ideas available for instruction? In the lesson, are modifications made when it appears that students are bored (the tasks are too easy), or lost (the tasks are too difficult)? Are there ways to design for such circumstances in advance?

We begin each case study with an extended exploration of the mathematics involved. What opportunities does the task or problem provide for deep mathematical thinking? How many different ways are there to solve the problem? How can we connect the solutions, and what can we learn from those connections? After exploring the mathematics, we then describe the classroom in some detail – the students and the teacher, what the students are likely to know, and whatever background that readers might find useful when they think about the lesson. Then we examine the lesson. We stop every few minutes, to explore the possibilities as discussed above. (Of course, those questions are generic. In the case studies, we explore particulars.) At the end of each case, we review the lesson as a whole.

Each case is written as though we were in a room with the teachers, discussing the lesson. “How would you summarize what has happened so far? Does anything in particular strike you?” “We see the following dilemma. What do you think the options are, what are the possible consequences?” Then we say what we think – not because it's necessarily the “right answer,” but because it could serve as material for further conversation with the group. That's how we've done “in-person” professional development, and we've done our best to capture the character of those conversations. We think of the book as a conversation between us and a community of teachers. In fact, we hope that groups of teachers will read the book together, working through the cases.

Why? We hope that, once the community has learned to problematize teaching as suggested by the cases, it will continue to do so with their own teaching. In the successful learning communities we know, teachers bring issues to the group. They say “I'd like to work on this issue (or problem),” and tape their attempts. Then the group analyzes what happened, making suggestions for next time. Our goal in the book is to build the capacity of teachers to do this kind of inquiry, in an ongoing way. Ongoing reflection is the key to professional growth.

Of course, coming up with the “right” questions to stimulate rich discussions is not easy. For that reason, the book also contains two tools, the *TRU Conversation Guide* and the *TRU Observation Guide*. For each dimension, both guides offer questions that can be used to plan lessons, to keep in mind during the lessons, and to reflect on them. A wide range of groups have found these questions to be useful stimuli to professional growth.

Mathematics Teaching On Target: A guide to Teaching for Robust Understanding at all Grade Levels

The vast majority of curricular materials lack imagination or relevance to student lives, making it difficult for students to have rich and meaningful conversations in class. The question is: can we enrich the tasks or activities, to provide more opportunities for ambitious and equitable instruction?

The TRU Framework provides a structure for thinking about this question. We can ask about how to: (1) enrich the mathematics in the task or activity; (2) modify the task, so that more students have the opportunity to engage in productive struggle; (3) open up the task, perhaps to different approaches, so that more students can engage meaningfully; (4) provide more opportunities for students to discuss and explain their work to each other, and (5) make student thinking visible, so that instruction can be modified for greater impact on student learning.

The metaphor we use is a target. When you shoot an arrow at a target, you hope that it will land near the center. Now consider an *educational* target. If a task “lands” near the center of the target, then it will provide rich opportunities for learning. If a task is not near the center, then the question is, what can we do to move it toward the center?

Our new book *Mathematics Teaching On Target: A guide to Teaching for Robust Understanding at all Grade Levels* (Schoenfeld, Fink, Sayavedra, Weltman, & Zuñiga-Ruiz, 2023) offers fifteen targets – three targets for each of the five dimensions. Here we describe the mathematics targets and give one example of their use.

For the mathematics dimension, our over-arching question is,

In what ways do classroom activities provide opportunities for students to become knowledgeable, flexible, and resourceful mathematical thinkers?

This big question is decomposed into three main sub-questions:

Sub-Question 1: What is the main mathematical idea? How does it develop? How is it connected to what students know? How is it connected to the grade level standards for mathematics content and practices?

Sub-Question 2: In what ways does student participation in classroom activities support their learning of mathematical content? What connections are built between procedures, underlying concepts, and meaningful contexts of application?

Sub-Question 3: In what ways does student participation in classroom activities support the development of important mathematical practices and other productive mathematical habits of mind?

Each sub-question serves as the main topic for one target. As an illustration, Figure 3 shows the first mathematics target, which corresponds to sub-question 1 above. You will find

many properties of tasks on the target. The properties on the outer ring provide little opportunity for students to have deep mathematical conversations. There are more opportunities in the middle ring; and tasks that have the properties on the inner circle can stimulate rich and meaningful mathematical discussions. The idea is to look at a task and see if you can make modifications that “move” it more to the center of the target. Many modifications are possible for each target – and remember, we have 15 targets!

The Mathematics **1. What is the main mathematical idea? How does it develop? How is it connected to what students know? How is it connected to the grade level content and practice standards?**

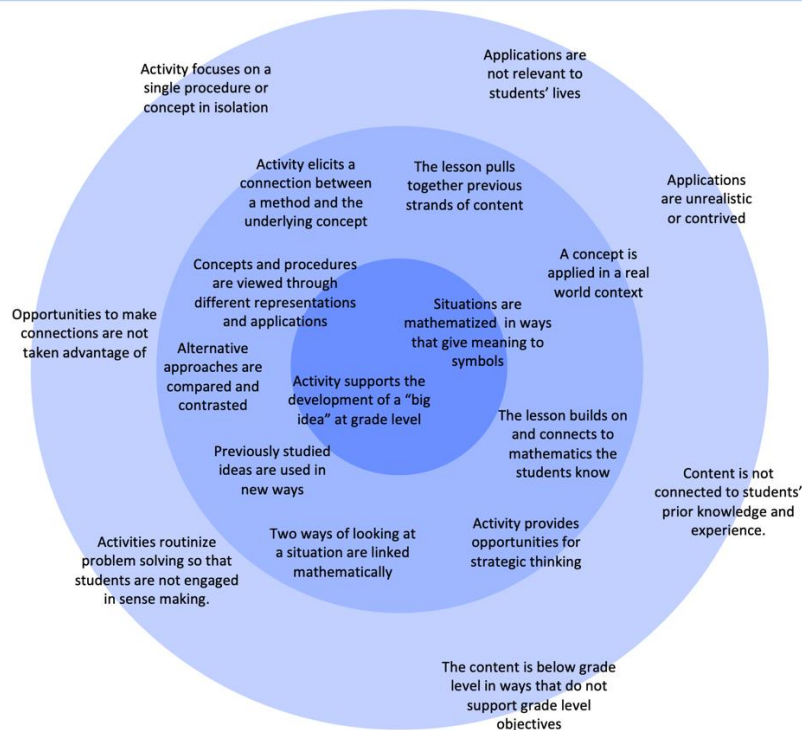


Figure 3. The first mathematics target.

In *On Target*, we work through many examples of how tasks can be improved. There are three examples for each dimension – one each at the elementary, middle school, and secondary school levels. Here is just one example from the book.

The topic we consider is exponential decay. If you google “exponential decay problems worksheet,” you will find more than half a million results. Most of them contain problems like this:

Over the past few years, the number of students enrolled in after-school programs has been decreasing. Each year there is a 11% decrease in student enrollment. Currently, 13,145 students are enrolled. If this trend continues, how many students will be enrolled in 6 years?

This task is neither realistic nor interesting. As we note in *On Target*, “The solution offered is a purely mechanical plug-in to the formula, $FV = PV(1-d)^n$, where FV = future value, PV = present value, d = rate of decay, and n = number of periods. There are many exercises of this type. What can we do to enrich them?”

To do so we look at the three mathematics targets. We identify the following mathematics challenges (properties of the task that were on the outer rings):

- Application is not relevant to students’ lives
- Application is unrealistic or contrived
- Content is not connected to students’ prior knowledge and experience
- Applications such as word problems are formulaic and not tied to making sense of the contexts
- Formulaic problem solutions provide little opportunity for sense making
- Students work on tasks just like the ones they’ve seen solved

But, when we look at the inner rings, we see possible directions for improvement:

- The lesson pulls together previous strands of content
- A concept is applied in a real world context
- Situations are mathematized in ways that give meaning to symbols
- Students compare and contrast different approaches to a problem
- Students work problems that extend what they know
- Students observe/derive properties of mathematical objects
- Students discuss and evaluate each other’s approaches and ideas
- Students build and evaluate models of real world situations
- Students look for patterns, making and testing conjectures

There are many ways we could try to enrich the original problem using these ideas. In fact, this example is real: one of the authors of *Mathematics Teaching on Target* created the following task for her students:

Anay buys a car for \$5,000. The car loses 15% of its value every year.

- a. How much is the car worth after 1 year?
- b. Write an equation to model the value of the car over time.

Before you go on, find a way to check/justify that your equation is realistic and show your work.

- c. How long before the car is worth half of its original value?
- d. After owning the car for 10 years, it breaks down. Anay finds out that she will need to replace the clutch to be able to drive the car again. Is it worth it?

This task is *much* richer than the enrollment problem above. The classroom discussion of this problem is one of the case studies in Schoenfeld, Fink, Ruiz, Huang, Wei, & Chirinda (2023). In that case study you’ll see the impact of the suggested modifications.

Discussion

In this brief paper I have tried to summarize nearly 20 years of research and development, including the content of two new books. There was very little space to provide detail. In my presentation at CIAEM I will provide more examples. Detail substantiating the claims made here can be found in the papers on the TRU Framework website, <https://truframework.org/>. Very substantial detail can be found in the two new books, *Helping students become powerful mathematics thinkers: Case Studies and Methods of Teaching for Robust Understanding* (Schoenfeld, Fink, Zuñiga-Ruiz, Huang, Wei, & Chirinda, 2023) and *Mathematics Teaching On Target: A guide to Teaching for Robust Understanding at all Grade Levels* (Schoenfeld, Fink, Sayavedra, Weltman, & Zuñiga-Ruiz, 2023). I hope you will find them worth investigating.

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